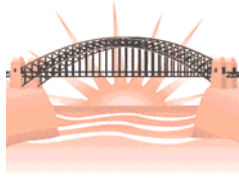




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## **Estimation of Fractal Dimension: Numerical Experiments and Software**

On this Web page we present results of our cooperative research aiming at computation of fractal dimension. You can find here a few papers, references and corresponding software to simulate fractals and estimate fractal dimension.



- ▲ The software is free. Please, refer to the software when you use it in your work.
- ▲ Please, inform us about successful application of the software.
- ▲ The software is still under development. Any feedback is highly appreciated. Please, send your remarks to Sergei M. Prigarin [\[send e-mail\]](#) or Klaus Hahn [\[send e-mail\]](#)
- ☼ The research was partially supported by RFBR (11-01-00641).

## Software

Programs to estimate fractal dimension of a time series, MS Win binaries - [read a brief description and download](#)

Simulation of the fractional Brownian motion (the generalized Wiener process): PASCAL source code and MS Win binary file - [read a brief description and download](#)

Simulation of random fields on the plane with different fractal dimensions (including one moving average scheme and fractional Brownian motion and sheet): MS Win binary files - [read a brief description and download](#)

Estimation of the fractal dimension by the variance counting method for a random function defined on the plane, MS Win binary file - [read a brief description and download](#)

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Programs to estimate fractal dimension of 3D sets. Simulation of the Menger sponge (3D analog of the Sierpinski carpet) and the fractal foam: MS Win binary files - [read a brief description and download](#)

## References

**Prigarin, S. M., Hahn, K., and Winkler, G. (2007) Evaluation of two numerical methods to measure the Hausdorff dimension of the fractional Brownian motion, Preprint 07-01, IBB, GSF Neuherberg, 28p.**

[http://www.helmholtz-muenchen.de/fileadmin/IBB/PDF/Research/Preprints/Preprints\\_2007/pp07-01.pdf](http://www.helmholtz-muenchen.de/fileadmin/IBB/PDF/Research/Preprints/Preprints_2007/pp07-01.pdf)

**Abstract** — The aim of the paper is to study by Monte Carlo simulation statistical properties of two numerical methods (the extended counting method and the variance counting method) developed to estimate the Hausdorff dimension of a time series and applied to the fractional Brownian motion.

**Key words:** fractal set, Hausdorff dimension, extended counting method, variance counting method, generalized Wiener process, fractional Brownian motion.

**Mathematical Subject Classification:** 28A80, 62M10, 65C05.

**Prigarin, S. M., Hahn, K., and Winkler, G. (2008) Comparative analysis of two numerical methods to measure Hausdorff dimension of the fractional Brownian motion, Numerical Analysis and Applications, V.1, No.2, P.163-178**

**Abstract** — Using Monte Carlo simulation techniques, we look at statistical properties of two numerical methods (the extended counting method and the variance counting method) developed to estimate the Hausdorff dimension of a time series and applied to the fractional Brownian motion.

**DOI:** 10.1134/S1995423908020079

**Key words:** fractal set, Hausdorff dimension, extended counting method, variance counting method, generalized Wiener process, fractional Brownian motion.

**Prigarin, S. M., Hahn, K., and Winkler, G. (2009), Variational dimension of random sequences and its application, Numerical Analysis and Applications, V.2, No.4, P.352-363**

**Abstract** — A concept of variational dimension is introduced for a random sequence with stationary increments. In Gaussian case the variational dimension in the limit coincides with the Hausdorff dimension of a proper random process. Applications of the concept are illustrated by examples of neurological data and network traffic analysis.

**Key words:** random sequences with stationary increments, variational dimension, Hausdorff dimension, fractal, self-similarity, data analysis.

**Mathematical Subject Classification:** 28A80, 62M10, 65C05.

**Prigarin, S. M., Hahn, K., and Winkler, G. (2010) On variational dimension of random fields, Preprint 10-01, IBB, GSF Neuherberg, 16p.**

[http://www.helmholtz-muenchen.de/fileadmin/IBB/PDF/Research/Preprints/Preprints\\_2010/pp01-10.pdf](http://www.helmholtz-muenchen.de/fileadmin/IBB/PDF/Research/Preprints/Preprints_2010/pp01-10.pdf)

**Abstract** — A concept of variational dimension is introduced for random fields. For a class of Gaussian fields with homogeneous increments the variational dimension converges to

the Hausdorff dimension. Several examples are presented to illustrate that the concept of variational dimension can be used to construct effective numerical methods to study fractal properties of random fields.

**Key words:** random fields, variational dimension, Hausdorff dimension, self-similarity, fractal analysis.

**Mathematical Subject Classification:** 28A80, 60G60, 65C05.

**Prigarin, S. M., Hahn, K., and Winkler, G. (2011) Estimation of fractal dimension of random fields on the basis of variance analysis of increments, Numerical Analysis and Applications, V.4, No.1, P.71-80**

**Abstract** — This paper deals with estimation of fractal dimension of realizations of random fields. Numerical methods are based on analysis of variance of increments. It is proposed to study fractal properties with the use of a specific characteristic of random fields called a “variational dimension.” For a class of Gaussian fields with homogeneous increments the variational dimension converges to the Hausdorff dimension. Several examples are presented to illustrate that the concept of variational dimension can be used to construct effective computational methods.

**Prigarin, S. M., Sandau, K., Kazmierczak, M., and Hahn, K. (2012) Estimation of fractal dimension: a survey with numerical experiments and software description, International Journal of Biomathematics and Biostatistics (in press)**

<http://www.helmholtz-muenchen.de/ibb/homepage/klaus.hahn/fractal/dim2012.pdf>

**Abstract** — A unifying discussion of basic concepts for the numerical estimation of fractal dimensions and multifractal spectra is presented. For fractal dimensions focus is on box-counting methods, multifractal spectra are introduced by generalized dimensions. Due to limited resolution of empirical data, a discrepancy between analytical and numerical estimation of fractal dimensions can be observed, the corresponding problem of validation is discussed. Numerical estimation is supported by implemented software, which can be downloaded. Numerical experiments are presented and simplify the application of the software, which contains numerous examples.

**Key words:** fractal dimension, box-counting method, extended counting method, generalized dimensions, multifractal spectrum, numerical methods and software.

**Mathematical Subject Classification:** 28A80, 65Z05.

# A Brief Software Description

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## Programs to estimate fractal dimension of a time series, [MS Win binary files](#)

You can find 2 programs in the [downloaded file](#) to estimate fractal dimension of a time series and examples of time series with fractal dimensions 1.1, 1.3, 1.5, 1.7, 1.9 to test the programs. Two methods are realized in the programs: variance counting and extended box counting methods, see details and further references in Prigarin S.M., Hahn K., and Winkler G. (2007, 2008, 2009).

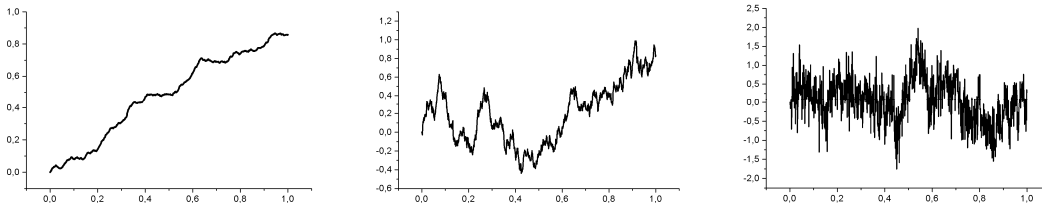
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## Simulation of the fractional Brownian motion (the generalized Wiener process): [MS Win binary file and PASCAL source code](#)

Fractional Brownian motion is a continuous Gaussian random process  $w(t)$  with mean zero and correlation function

$$E[w(t) w(s)] = 0.5 (|t|^a + |s|^a - |t-s|^a),$$

where  $a$  is a parameter in  $(0,2]$ . If  $a=1$  then the process is the “ordinary” Brownian motion with independent increments. The Hausdorff dimension of the fractional Brownian motion with parameter  $a$  is equal to  $2-a/2$ . Realizations of the fractal Brownian motion with dimensions 1.1, 1.5, 1.9 are shown below:



In the [presented software](#), the simulation of the fractional Brownian motion is based on the Cholesky decomposition of the correlation matrix (it is an exact but slow and storage-consuming method for Gaussian vectors).

Simulation of random fields on the plane with different fractal dimensions (including one moving average scheme, fractional Brownian motion and sheet): [MS Win binary](#)

**Program FBsim.EXE:** Aim of the program is to simulate realizations of the fractional Brownian motion and sheet. The simulated field will be written in ASCII file **FB.DAT**. The fractal dimension of the realizations (surfaces in 3D space) can vary from 2 to 3. In addition, the program will estimate fractal dimension of the simulated realization by 4 methods, according to formulas (12)-(15) from Prigarin S.M., Hahn K., and Winkler G. (2011). In the output text the dimensions will be called:

*DimEst1*, see formula (12),

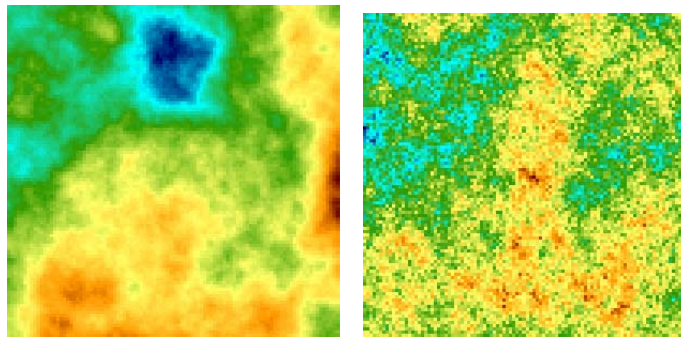
*DimEst2*, see formula (13),

*DimEst*, see formula (14),

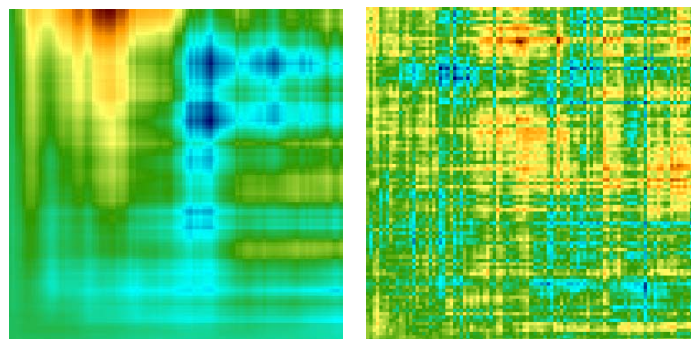
*DimEst0*, see formula (15); this estimator is not appropriate for the fractional Brownian sheet.

The simulation algorithm is based on the Cholesky decomposition of the correlation matrix (it is an exact but slow and storage-consuming method for Gaussian distributions).

Below we present realizations of fractional Brownian motion:

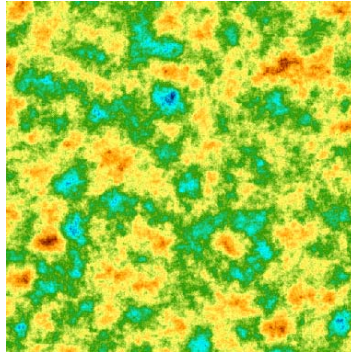


and fractional Brownian sheet:



with the Hausdorff dimensions 2.2 (on the left) and 2.8 (on the right).

**Program MA2iso.EXE:** Aim of the program is to simulate realizations of an isotropic Gaussian random field on the plane by moving average scheme, see details and further references in Prigarin S.M., Hahn K., and Winkler G. (2010, 2011). The Hausdorff dimension of the realizations is 2.5. The simulated field will be written in ASCII file **MA2iso.DAT**. A realization of the field is presented below:



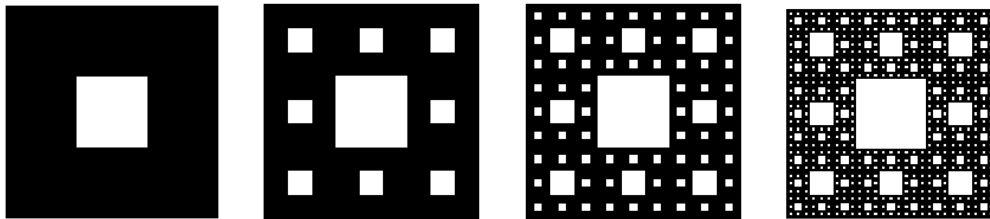
### **Estimation of fractal dimension by the variance counting method for a random function defined on the plane: [MS Win binary file](#)**

Program **Get-vDim-2D.EXE** is designed to estimate variance dimension of a surface, given as a matrix in an ASCII file. In addition, there are examples of input files **FB.DAT** (with a realization of the 2D fractional Brownian motion, dimension 2.75) and **MA2iso.DAT** (with a realization of a Gaussian isotropic homogeneous field, dimension 2.5). Files **FB.DAT** and **MA2iso.DAT** were generated by programs **FBsim.EXE**, **MA2iso.EXE** (see the above software description). Variance dimension is estimated by 4 methods, according to formulas (12)-(15) from Prigarin S.M., Hahn K., and Winkler G. (2011): in the output of the program the dimensions will be called *DimEst1* (corresponds to formula (12)), *DimEst2* (corresponds to formula (13)), *DimEst* (corresponds to formula (14)), *DimEst0* (corresponds to formula (15)).

Simulation of several 2D fractals (the Sierpinski carpet, the Cantor dust, the fractal percolation model, the Henon attractor), [MS Win binary files](#) :

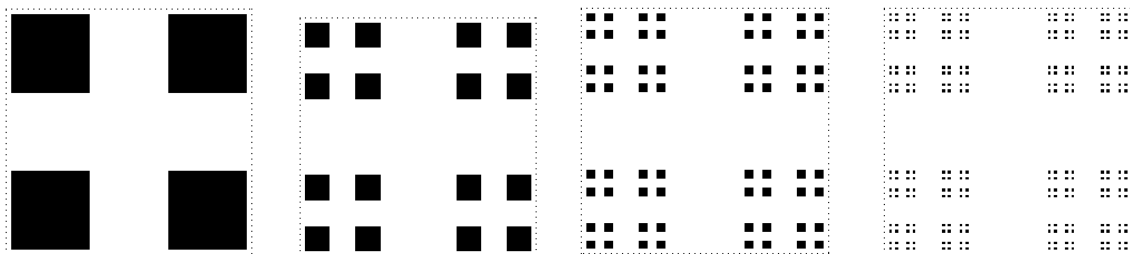
**Sim-Sierpinski2D.EXE, Sierpinski2D\_XY.EXE** : the Sierpinski carpet, dimension =  $\ln(8)/\ln(3) = 1.893$ . The output for **Sim-Sierpinski2D.EXE** is ASCII file **Sim-Sierpinski2D.MAT** with a matrix of elements 0 and 1 (0 corresponds to parts without points of the Sierpinski carpet). The output for **Sierpinski2D\_XY.EXE** is ASCII file **Sierpinski2D.XY** with (X,Y)-coordinates of points of the Sierpinski carpet.

*First 4 iterations of the Sierpinski carpet:*



**Sim-CantorDust2D.EXE** : the Cantor dust, dimension =  $\ln(4)/\ln(3) = 1.262$ . The output is ASCII file **Sim-CantorDust2D.MAT** with a matrix of elements 0 and 1 (0 corresponds to parts without points of the Sierpinski carpet).

*First 4 iterations of the Cantor dust:*



**Sim-StochCarpet2D.EXE** : stochastic modification of the Sierpinski carpet. The output is ASCII file **Sim-StochCarpet2D.MAT** with a matrix of elements 0 and 1 (0 corresponds to parts without points of the stochastic carpet). Input parameters: basic grid dimension (BGD) is a number of intervals on the square side for the first iteration (i.e. the square is divided into  $BGD*BGD$  subsquares), number  $M$  of nonempty subsquares in every square. The dimension of such a fractal is  $\ln(M)/\ln(BGD)$ .



*A realization of stochastic carpet with parameters (BGD=4, M=14):*



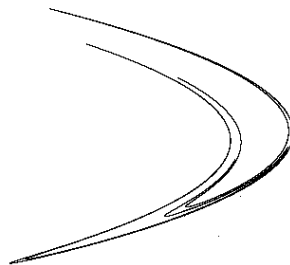
**Sim-Percol2D.EXE** : 2D fractal percolation model. The model is described, for example, in K.Falconer, Fractal Geometry, 2003 (see p.251)

*A realization of the 2D fractal percolation model:*



**Henon\_XY.EXE** : the canonical Henon attractor with a Hausdorff dimension  $\approx 1.261 \pm 0.003$  and a correlation dimension  $\approx 1.25 \pm 0.02$ . (X,Y)-coordinates of points of the canonical Henon attractor will be written in ASCII file **Henon\_canonical.XY**.

*The canonical Henon attractor:*



**Henon\_attractor.EXE** : the Henon attractor with given parameters. The parameters and number of points to be simulated must be presented in file **Henon\_attractor.INP**. (X,Y)-coordinates of points of the Henon attractor will be written in ASCII file **Henon\_attractor.XY**.

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## Programs to estimate fractal dimension of 2D sets, [MS Win binaries](#)

Program **xDim\_2D.EXE** : computation of the fractal dimension of a 2D set by the *extended counting method*.

Program **012-Dim\_2D.exe** : computation of *two-scale* box, information, and correlation dimensions of a 2D set.

Program **gen-Dim-Tab\_2D.exe** : computation of a table with the *two-scale* generalized dimensions of a 2D set.

Program **GenDim2D.EXE** (with input files **GenDim2D.INP**, **GenDim2D.MAT** and an example of command file **GenDim2D.BAT**): computation of a table with the generalized dimensions of a 2D set by least squares. *Adjusted* generalized dimensions are computed as well.

In addition, examples of 2D fractal sets are presented in files CANTORDUST2D.MAT, PERCOL2D.MAT, SIERPINSKI2D.MAT, STOCHCAPERT2D.MAT.

Utility **VisuFish.EXE** (with file **VisuFishHelp.TXT**) can be used to visualize MAT-files.

About *extended counting*, *2-scale* and *adjusted* methods see Prigarin S.M., Hahn K., and Winkler G. (2001b) and references there.

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## Programs to estimate fractal dimension of 3D sets. Simulation of the Menger sponge (3D analog of the Sierpinski carpet) and the fractal foam, [MS Win binary files](#)

**Menger-sponge.EXE** : the Menger sponge with  $\text{dimension} = \ln(20)/\ln(3) = 2.7268$  (output will be in file **Menger-sponge.XYZ** with 3 columns of (x,y,z)-coordinates of the set points)

*First 3 iterations of the Menger sponge:*



**Fractal-Foam.EXE** : the fractal foam with dimension= $\ln(26)/\ln(3)= 2.9656$  (output will be in file **Fractal-foam.XYZ** with 3 columns of (x,y,z)-coordinates of the points), the construction of the fractal foam is similar to that for the Menger sponge.

**Show3Dxyz.EXE** : a simple utility to visualize XYZ-files

**x-Dim3D.EXE** : computation of fractal dimension by the extended counting method of a set presented in ASCII file **x-Dim3D.XYZ**

**box-Dim3D.EXE** : computation of box-dimensions (and adjusted box-dimensions) of a set presented in ASCII file **box-Dim3D.XYZ**. The results will be written in files **box-Dim3D.REZ**, **box-Dim3D.LN**

Additional details can be found in Prigarin S.M., Hahn K., and Winkler G. (20011b) and references there.



## A few additional Links

[Fractal analysis system for Windows](#)

[Fractal Dimension Calculator](#)

[A fractal analysis toolbox for Matlab](#)

[Fractal Analysis Programs](#)