MONTE CARLO SIMULATION OF 3D RADIATION TRANSFER IN WATER-DROP AND CRYSTAL CLOUDS WITH ALLOWANCE FOR POLARIZATION AND OPTICAL ANISOTROPY

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3 in 1

1. Monte Carlo simulation of radiation transfer in optically anisotropic media
2. Hypothetic “fractal anisotropy” in clouds
3. Monte Carlo simulation of angular characteristics for polarized radiation in water-drop and crystal clouds
The main objective of the research: to develop a mathematical model and a Monte Carlo algorithm to simulate the radiation transfer in a scattering medium that is optically anisotropic with respect to the zenith angle of a light beam.
Monte Carlo simulation of photons trajectories in a scattering medium that is optically anisotropic with respect to the zenith angle

- Step 1. An initial point \( r_0 = (x_0, y_0, z_0) \) and an initial direction \( \omega_0 = (a_0, b_0, c_0) \), \( |\omega_0| = 1 \) of a photon are simulated according to the density of sources; \( n=0 \).
- Step 2. A photon free-path length \( l \) is simulated according to the probability density

\[
p(l) = \sigma(c_n) \exp\{-l \sigma(c_n)\}, \quad l > 0,
\]

where \( \sigma(c) \) is the extinction coefficient in the direction with cosine \( c \) of the zenith angle.
- Step 3. We set \( n = n+1 \) and calculate the coordinates of the next collision point in the medium:

\[
x_n = x_{n-1} + a_{n-1} l, \quad y_n = y_{n-1} + b_{n-1} l, \quad z_n = z_{n-1} + c_n l, \quad r_n = (x_n, y_n, z_n).
\]
- Step 4. The type of collision is simulated: it is scattering with probability \( q(c_n) \) and absorption with probability \( 1 - q(c_n) \), where \( q(c) \) is a single scattering albedo in the direction with cosine \( c \) of the zenith angle. The steps below are carried out only in the case of scattering.
- Step 5. A new direction of the photon \( \omega_n = (a_n, b_n, c_n) \) is simulated according to the phase function \( g(\omega_{n-1}, \omega_n) \). Go over to step 2.
Denote the direction of a photon before scattering by \( \omega' = (a', b', c') \) and the direction after scattering by \( \omega = (a, b, c) \). First, the value of cosine \( c \) of the zenith angle is simulated. For that, we use a distribution \( P(c', c) \) with respect to \( c \in [-1, 1] \) when the value \( c' \) is fixed. Then a variation \( \psi \) of the azimuthal angle is simulated. In this case we need to know a distribution \( Q(c', c, \psi) \) with respect to \( \psi \in [-\pi, \pi] \) when the values \( c', c \) are fixed. The values \( a \) and \( b \) can be found by the formulas

\[
\begin{align*}
a &= [a' \cos(\psi) - b' \sin(\psi)] (1 - c'^2)^{1/2} [1 - (c')^2]^{-1/2}, \\
b &= [a' \sin(\psi) - b' \cos(\psi)] (1 - c'^2)^{1/2} [1 - (c')^2]^{-1/2}.
\end{align*}
\]

Thus, for the anisotropic scattering model it is necessary to know the families of distributions \( P(c', \cdot), Q(c', c, \cdot) \) where \( c' \in [-1, 1], c \in [-1, 1] \).
3 ice cloud models

1. Columns, stochastically oriented in space

2. Horizontally oriented columns

3. Parry columns

Half-height/edge = 2.5, refractive index = 1.311

The optical properties of the scattering media with ice crystals were computed on the basis of a pure geometric optics approach.

Monte Carlo simulation of ice cloud halos
Optical depth of the cloud: 3; Source zenith angle: 0°

1. Columns, stochastically oriented in space
2. Horizontally oriented columns
3. Parry columns
Monte Carlo simulation of ice cloud halos

Optical depth of the cloud: 3; Source zenith angle: 45°

1. Columns, stochastically oriented in space
2. Horizontally oriented columns
3. Parry columns
Monte Carlo simulation of ice cloud halos
Optical depth of the cloud: 3; Source zenith angle: 75°

1. Columns, stochastically oriented in space
2. Horizontally oriented columns
3. Parry columns
Albedo for the 3 models of ice cloud (optical depth = 3, columns)
Conclusion:

A new Monte Carlo algorithm was developed to simulate the radiation transfer processes in a medium, optically anisotropic with respect to the zenith angle of a photon. Preliminary numerical experiments show that it is very important to take into account the orientation of the ice particles, and that the anisotropy of a medium can strongly affect the optical properties of crystal clouds.
Part 2

**Hypothesis:** the optical anisotropy of clouds in the atmosphere can follow not only from the shape and the orientation of scattering particles, but furthermore, it can be a result of a random non-Poisson distribution of particles in space. In that way even for water-drop clouds, the optical medium can be appreciably anisotropic if the spherical water drops a specifically distributed in space. An admissible model for the spatial distribution can be obtained under assumption that particles are concentrated on a manifold of a dimension less than 3. This kind of the optical anisotropy in scattering media will be called «fractal anisotropy». 
The following observations are the basis of the hypothesis:
- distribution of scattering particles in a cloud can be non-Poisson
- the fractal nature of spatial distributions for scattering particles in a cloud

Publications on distribution of scattering particles in clouds:


Poisson distribution

Non-Poisson anisotropic distributions:
A model of “fractal anisotropy”

- **Optically isotropic homogeneous media:**
  - phase function $g$
  - single scattering albedo $q$
  - extinction coefficient $\sigma$
- **Fractal anisotropy:**
  - phase function $g$
  - single scattering albedo $q$
  - extinction coefficient in the direction $\omega=(a,b,c)$:
    $$\sigma(\omega) = \sigma / \{(a/c_x)^2 + (b/c_y)^2 + (c/c_z)^2\}^{1/2}, \omega=(a,b,c), c_x c_y c_z = 1$$
• The range of a particle from the point \((0,0,0)\) to the point \((x,y,z)\) in the optically isotropic medium corresponds to the range from the point \((0,0,0)\) to the point \((c_x x, c_y y, c_z z)\) in the anisotropic medium. Here \(c_x, c_y, c_z\) are compression coefficients for the compression directions \(OX, OY, OZ\), \(c_x c_y c_z = 1\).

• In the general case, there are 3 orthogonal compression directions with unit vectors \(e_1, e_2, e_3\) and compression coefficients \(c_1, c_2, c_3\), \(c_1 c_2 c_3 = 1\). In this case

\[
\sigma(\omega) = \sigma / :T^{-1} \omega :,
\]

where \(T\) is the compression tensor:

\[
T = c_1 e_1 e_1^T + c_2 e_2 e_2^T + c_3 e_3 e_3^T
\]
Heuristic derivation of the model

1. A homogeneous optically isotropic scattering medium with extinction coefficient $\sigma$ is concentrated in a whole space volume without empty spaces.

2. A scattering medium with extinction coefficient $\sigma/P$ is concentrated on a random homogeneous isotropic set $S$ that covers a portion $P<1$ of the space volume. The linear sizes of empty regions are sufficiently small and uncorrelated.

3. $P$ goes to zero; hence, the measure of $S$ converges to zero as well ($S$ is a fractal).

4. Instead of the set $S=\{(x,y,z)\}$ we consider the set $TS=\{T(x,y,z): (x,y,z)\in S\}$, where $T$ is a compression tensor.
Numerical experiments (Monte Carlo simulation)

Which radiation effects can be caused by the fractal anisotropy?

For the numerical experiments we set:

\[ c_2 = c_3 = c_1^{-1/2} \]

\( c_1 \) is the \textit{basic compression coefficient}

\( e_1 \) is the \textit{basic compression direction}
- visible range of wavelength; SSA=1; C1 cloud layer;
- thickness: \textbf{250 m}; ext. coeff. = 0.02 m\(^{-1}\) (optical depth = 5)
- \(\mathbf{e}_1=(0,0,1)\): the basic compression direction is vertical
  \begin{itemize}
  \item \textbf{red}: optically isotropic medium, \(c_1=1\)
  \item \textbf{green}: vertically dense medium, \(c_1=4\)
  \item \textbf{black}: vertically rare medium, \(c_1=0.25\)
  \end{itemize}
- $e_1 = (\sin(Z), 0, \cos(Z))$, where $Z$ is the zenithal angle of the basic compression direction: $Z = 00, 45, 60, 90$ deg.
Angular distributions of the downward radiation (the sun is in the zenith)

Isotropic medium (optical thickness = 5)

Vertically dense medium: 
\[ e_1=(0,0,1), \ c_1=4 \]

Horizontally rare medium: 
\[ e_1=(1,0,0), \ c_1=0.25 \]
The aim of the research:
To study by Monte Carlo method the dependence of angular distributions for polarized radiation fields on optical properties of water-drop and crystal clouds.
Preliminary conclusions:

• Angular distributions for different radiation characteristics (intensity, polarization degree, preferable direction of polarization) supplement each other and enable us to define the particles size and the shape more precisely.

• For optically thick clouds, the angular distributions for the reflected radiation are more informative than the distributions for the transmitted radiation.
Angular distributions for the **upward** radiation, water-drop clouds, optical thickness=1, zenith angle=0°, wavelength=532nm

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albedo</td>
<td>4%</td>
<td>4.8%</td>
<td>6%</td>
</tr>
</tbody>
</table>
Degree of polarization for the upward radiation distributed on the dome of the sky, water-drop clouds, optical thickness=1, zenith angle=0°, wavelength=532nm

C1                                C2                       C3
Max: 0.59                        0.48                           0.37
Degree of polarization for the downward radiation distributed on the dome of the sky, water-drop clouds, optical thickness=1, zenith angle=0°, wavelength=532nm

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>0.15</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td>Sc. transmittance</td>
<td>59%</td>
<td>58%</td>
<td>58%</td>
</tr>
</tbody>
</table>
Size distributions of water drops in clouds C1, C2, C3
(D. Deirmendjian, Electromagnetic Scattering on Spherical Polydispersions, 1969)
Angular distributions for the **upward** radiation, water-drop clouds, optical thickness=1, zenith angle=30°, wavelength=532nm

C1
Albedo: 5.5%

C2
6.4%

C3
8%
Degree of polarization for the upward radiation distributed on the dome of the sky, water-drop clouds, optical thickness=1, zenith angle=30°, wavelength=532nm
Degree of polarization for the downward radiation distributed on the dome of the sky, water-drop clouds, optical thickness=1, zenith angle=30°, wavelength=532nm.

Max: 0.27 0.21 0.12
Angular distributions for the **upward** radiation, water-drop clouds, optical thickness=10, zenith angle=30°, wavelength=532nm
Degree of polarization for the upward radiation distributed on the dome of the sky, water-drop clouds, optical thickness=10, zenith angle=30°, wavelength=532nm

C1                               C2                       C3
Max: 0.33                        0.21                           0.14
Angular distributions for the **downward** radiation, water-drop clouds, optical thickness=10, zenith angle=30°, wavelength=532nm

<table>
<thead>
<tr>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>transmittance: 55.8%</td>
<td>52.3%</td>
<td>47.3%</td>
</tr>
</tbody>
</table>
Angular distributions for the **upward** radiation, ice clouds, optical thickness=1, zenith angle=0°, wavelength=532nm

- oblate cylinders: 2.9%
- dendrite: 3.1%
- hexagonal column: 5.9%
- 4 bullet rosette: 4.7%
Degree of polarization for the upward radiation distributed on the dome of the sky, ice clouds, optical thickness=1, zenith angle=0°, wavelength=532nm

- oblate cylinders: max 0.12
- dendrite: 0.53
- hexagonal column: 0.55
- 4 bullet rosette: 0.55
Degree of polarization for the downward radiation distributed on the dome of the sky, ice clouds, optical thickness=1, zenith angle=0°, wavelength=532nm

- Oblate cylinders: max: 0.03, Sc. transmittance: 61%
- Dendrite: 0.29, 59%
- Hexagonal column: 0.15, 55.4%
- 4 bullet rosette: 0.21, 58%
Angular distributions for the **upward** radiation, ice clouds, optical thickness=1, zenith angle=30°, wavelength=532nm

- oblate cylinders: 4.5%
- dendrite: 4%
- hexagonal column: 7.7%
- 4 bullet rosette: 5.6%
Degree of polarization for the upward radiation distributed on the dome of the sky, ice clouds, optical thickness=1, zenith angle=30°, wavelength=532nm

- oblate cylinders: max = 0.12
- dendrite: 0.53
- hexagonal column: 0.55
- 4 bullet rosette: 0.55
Degree of polarization for the downward radiation distributed on the dome of the sky, ice clouds, optical thickness=1, zenith angle=30°, wavelength=532nm

- Oblate cylinders: max: 0.03, Sc. transmittance: 64%
- Dendrite: 0.29, 63%
- Hexagonal column: 0.15, 59%
- 4 bullet rosette: 0.21, 61.5%
Degree of polarization for the upward radiation distributed on the dome of the sky, ice clouds, optical thickness=10, zenith angle=30°, wavelength=532nm

Oblate cylinders: max = 0.17
Dendrite: 0.24
Hexagonal column: 0.29
4 bullet rosette: 0.18
Angular distributions for the **preferable polarization** (the angles are measured from the vertical plane) for the **upward** (on the left) and the **downward** (on the right) directions; ice cloud, hexagonal column, optical thickness=1, zenith angle=30°, wavelength=532nm
Subject for the future research:

Development of Monte Carlo code to simulate the radiation transfer processes for polarized light and optically anisotropic media.
Publications:


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http://alpha.emitter.googlepages.com/
http://osmf.sscce.ru/~smp