Spectral Models of Random Fields in Monte Carlo Methods

S.M.Prigarin

To my beloved Grannies,
Eugenia and Maria

Preface

The proposed spectral models represent a new class of numerical methods aimed at simulation of random processes and fields. The spectral models were developed in the 70-s, and they have appeared to be considerably promising for various applications. In Russia, the papers by Prof. G.A. Mikhailov have given impetus to a large number of investigations on spectral models of random fields. Nowadays the spectral models are extensively used for stochastic simulation in the atmosphere and ocean optics, turbulence theory, analysis of pollution transport for porous media, astrophysics, and other fields of science.

The scalar spectral models and some of their applications are presented in Chapter 1. A new technique of successive refinement of spectral models and conditional spectral models are described here. Chapter 2 deals with vector-valued spectral models. Convergence of spectral models is studied in Chapter 3. Problems of optimization and convergence for functional Monte Carlo estimators are considered in Chapter 4. Moreover, the monograph includes four Appendices, where some auxiliary information is presented and additional problems are discussed. Appendices deal with a) properties and methods of simulation of Gaussian distributions, b) numerical solution of boundary value problems for linear systems of stochastic differential equations, c) interpolation of stationary random sequences and their correlation functions, d) coding of multiplicative pseudo-random number generators in high-level programming languages.

The book is intended for experts in Monte Carlo methods, as well as for senior and post-graduate students, who are interested in the theory and applications of stochastic simulation. For preparing the text, the author used the notes of the lectures he has given at Novosibirsk State University. Knowledge of basic concepts of linear algebra, probability theory and functional analysis is desirable for reading the book. A number of “exercises” and “tasks” are offered for studying on reader’s own. Some of the “tasks” contain comparatively difficult and unsolved problems that can be used as subjects of graduate and post-graduate research.

The author would be interested in readers' references that can be sent at the following address or by electronic mail: Institute of Computational Mathematics and Mathematical Geophysics, pr. Lavrentieva, 6, Novosibirsk, 630090, Russia (Email: smp@osmf.sscc.ru).

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